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LETTER TO THE EDITOR

On multidimensional time

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Abstract. We discuss the relationship of space–time with three spatial and $2n+1$ temporal dimensions, which may be of interest in the theory of electroweak interactions, to ordinary four-dimensional space–time.

Up to now, the study of six-dimensional space–time with three spatial and three temporal dimensions has been restricted mainly to attempts at generalising special relativity to superluminal velocities (Mignani and Recami 1977, Cole 1978, Pappas 1978, Ziino 1979a, b, Cole 1980a, b). Now, however, Taylor has derived a result which may change this situation. He embedded the Salam–Weinberg electroweak theory in the graded algebra SU(2|1) (the smallest simple graded algebra containing SU(2)× U(1)) and predicted the Weinberg angle of 30° in good agreement with experimental data. Considering the electron as the only lepton, two extra time dimensions had to be included to get the right sign for the $(\text{mass})^2$ term of the Higgs scalar. For each further lepton two more time dimensions had to be added (Taylor 1980).

If Taylor's idea should prove useful, time with $2n+1$ dimensions ($n=0, 1, 2, \ldots$) may eventually become fashionable. Then the question would arise, whether this may have implications on the macroscopic scale. Let us sketch a possible answer.

Of all proposed six-dimensional schemes only Cole's second one (1980a, b) seems to be viable. Others predict results manifestly at variance with experimental data (Strnad 1978, 1979a, b). Cole introduces six-dimensional world-vectors $x^{\mathrm{T}}=(\boldsymbol{x}, \boldsymbol{t})$ ($c=1$ and T is the transpose) and demands the invariance of $x^{\mathrm{T}}Gx$. Here G is a 6×6 matrix with elements $+1, +1, +1, -1, -1, -1$ along the diagonal and zero elsewhere. The matrix of the Lorentz transformations is written in the form

$$L=\begin{pmatrix} A & P \\ Q & R \end{pmatrix}$$

with 3×3 matrices A, P, Q and R. From $L^{\mathrm{T}}GL=kG$, with $k=1$ in the subluminal and $k=-1$ in the superluminal case, it follows that

$$L^{-1}=k\begin{pmatrix} A^{\mathrm{T}} & -Q^{\mathrm{T}} \\ -P^{\mathrm{T}} & R^{\mathrm{T}} \end{pmatrix}.$$

Introducing two inertial reference frames, S with origin O and S' with origin O′, Cole derived relations among the matrices A, P, Q and R and expressed them finally in terms of three-dimensional velocities and three-dimensional time unit vectors of both origins.

He proved furthermore that all results of this scheme go over into corresponding results of four-dimensional relativity if the time unit vectors are parallel.

In the present situation Cole's argument can be reversed. If the results of four-dimensional relativity are valid, in the six-dimensional scheme all time vectors are parallel. This ensures that Taylor's idea with the electron as the only lepton has no implications in the macroscopic domain.

The use of Cole's argument may not even be necessary. Cole in his considerations has not exploited the reciprocity of reference frames, based on the relativity principle. In the derivation of the four-dimensional Lorentz transformation, this is stated in the following form: origin O has velocity $\boldsymbol{v}' = -\boldsymbol{v}$, measured in S', if origin O′ has velocity $\boldsymbol{v}$, measured in S. If reciprocity is demanded also for corresponding time unit vectors of origins O and O′, the six-dimensional Lorentz matrix L is symmetrical, as in the four-dimensional case: $A^{\mathrm{T}} = A$, $R^{\mathrm{T}} = R$, $P^{\mathrm{T}} = Q$. This in turn excludes $k = -1$, and therewith the possibility that in six dimensions a time-like vector is transformed into a space-like one and vice versa. This may limit the applicability of six-dimensional schemes in superluminal generalisations, but it is encouraging with respect to Taylor's idea. So the symmetry of the six-dimensional Lorentz matrix under usual conditions, being generalised versions of corresponding conditions in the ordinary four-dimensional theory, suffices to make time unit vectors parallel, without imposing explicitly the validity of the whole four-dimensional theory. Minimal conditions to ensure this are currently being investigated.

The sketched procedure is a straightforward generalisation of the usual technique in four dimensions. It can be further generalised to a space–time with $2n+1$ temporal dimensions leading to the same result. Only, for $n \neq 1$ P and Q are not square matrices. So Taylor's theory can be formulated in a space–time with multidimensional time without affecting well known results of four-dimensional special relativity (i.e. the classical approximation of quantum mechanics, e.g. particle trajectories in a bubble chamber).

While the proposed answer may be satisfactory from the macroscopic point of view, it deserves further study on a microscopic scale. Let us make in this context only two remarks. It is possible that a free particle trajectory in six-dimensional space–time is represented as a hypertube with respect to the three temporal dimensions, the time unit vectors of all tube axes being parallel. Superluminal propagation of signals is confined to the interior of the tubes. Some time ago, Terletskii (1968) suggested that superluminal propagation is restricted to fluctuations. The diameter of the tubes represents a 'transversal' fundamental time. Occasionally its 'longitudinal' counterpart has been advocated (Ehrlich 1977). Three-dimensional time, being of significance on the extreme microscopic scale only, would not be isotropic. Indeed, it is frequently assumed that space–time on this scale may not be homogeneous and isotropic (e.g. Blokhintsev 1964, Kim 1975, Cheon 1979).

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